## STUDENT NUMBER:

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The Irving K. Barber School of Arts and Sciences

Physics 231— Winter 2012/2013 - Term 1
FINAL EXAMINATION

Instructor: Jake Bobowski
Saturday, December 8, 2012 Time: 13:30-16:30
Location: ART 374

This Examination was prepared by Jake Bobowski
Not including this coversheet, the exam consists of 11 numbered pages.

- Attempt all of problems 1 through 4.
- Attempt any three of problems 5 through 8.

If necessary, you may use the backs of pages for calculations.
You must clearly show your work to receive full credit.
Writing down only the correct final answer will not earn full credit.
Include units with the final answer whenever appropriate.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 10 | 10 | 20 | 10 | 10 | 10 | 10 | 75 |
|  |  |  |  |  |  |  |  |  |

Free Response: Write out complete answers to the following questions. Show your work.
( $\left.5^{\text {pts }}\right)$ 1. Use Euler's equation $\left(e^{ \pm j \phi}=\cos \phi \pm j \sin \phi\right)$ to derive the following trigonometric identities:

$$
\begin{aligned}
\cos \left(\phi_{1} \pm \phi_{2}\right) & =\cos \phi_{1} \cos \phi_{2} \mp \sin \phi_{1} \sin \phi_{2} \\
\sin \left(\phi_{1} \pm \phi_{2}\right) & =\sin \phi_{1} \cos \phi_{2} \pm \cos \phi_{1} \sin \phi_{2}
\end{aligned}
$$

2. The quality factor $Q$ of an $L R C$-circuit is defined to be the ratio of the resonance frequency to the width of the resonance. For the parallel $L R C$-circuit, the $Q$-factor is given by:

$$
Q=R \sqrt{\frac{C}{L}}
$$

(a) If $R \pm \delta R, C \pm \delta C$, and $L \pm \delta L$ are determined by experimental measurements, what is the uncertainty in the calculated value of $Q$ ? Find an expression in terms of $R, \delta R, C, \delta C, L$, and $\delta L$.
(b) If $R$ and $C$ are both known to within $5 \%$ and $L$ is known to within $10 \%$, what is the percent uncertainty in $Q$ ?
(10 ${ }^{\text {pts }}$ ) 3. (a) Staring from $V_{C}=q / C$, and assuming time-harmonic (sinusoidal) signals, derive an expression for the impedance $Z_{C}$ of a capacitor.
(b) Staring from $V_{L}=L \frac{d i}{d t}$, and assuming time-harmonic (sinusoidal) signals, derive an expression for the impedance $Z_{L}$ of an inductor.
4. If we want to select signals within a range of frequencies from an input source which contains a wide distribution of frequencies (tuning a radio, for example), a resonant tank circuit can be used. In this problem, $i_{\mathrm{S}}$ is a current source.

(a) ( 5 pts ) Show that when $\omega \approx 1 / \sqrt{L C}$ the impedance of the parallel $L C$ combination is very large, such that the signal is passed to the receiver (resistor $R$ ). On the hand, show that when $\omega$ is far from $1 / \sqrt{L C}$, the impedance of the tank circuit is very low such that the signal is not passed to the receiver.
(b) ( 10 pts ) Recall from problem 2 that the width of the resonance, or equivalently, the range of frequencies selected by the tank circuit is determined by the $Q$ of the parallel $L R C$ circuit. To increase the selectivity of the tank circuit, we want to increase the $Q$ by using a sub-circuit between the tank and receiver to increase the effective resistance in parallel with the $L C$ combination.


The proposed sub-circuit is shown below. Determine the equivalent resistance $R_{\text {eq }}$ of this sub-circuit. That is, find the ratio $v / i \equiv R_{\text {eq }}$ where $v$ is the labelled voltage across the input terminals and $i$ is as shown in the figure. (Note that $v$ is measured between ground and the non-inverting input $v_{+}$of the op amp.) Hint: You should find that $R_{\mathrm{eq}}$ is negative!

(c) (5 pts) Finally, suppose $R=2000 \Omega$. What is the required value of $R_{\text {eq }}$ to make $R \| R_{\text {eq }}=$ $10 \mathrm{k} \Omega$ ? If $R_{1}=2 R_{3}$, what is the required value of $R_{2}$ ?

Complete any of the three remaining problems (5, 6, 7, 8).
Clearly indicate which three problems you which to be graded by entering three numbers into the table below.

(10 $\left.{ }^{\text {pts }}\right)$
5. (a) Consider the following five digital operations:

$$
Q=A \cdot B, \bar{Q}=\overline{\bar{A} \cdot \bar{B}}, Q=\overline{A \cdot B}, Q=\bar{A}+\bar{B}, Q=\bar{A} \cdot \bar{B}
$$

How many unique operations are represented in the list?
(b) Identify which of the operations listed in (a) are performed by the transistor circuit shown below? In the circuit below, lines that cross without a dot are not connected.

6. Calculate the Thevenin equivalent circuit parameters. Draw the Thevenin equivalent circuit.

7. In this circuit $V_{0}=410 \mathrm{~V}, R_{\mathrm{S}}=1200 \Omega, C=270 \mu \mathrm{~F}$, and $R=10 \Omega$. Assume that the switch has been positioned to the right (connecting $C$ and $R$ ) for a very long time.
(a) At $t=0$, the switch is flipped to the left. What is the voltage on the capacitor at $t=0.9 \mathrm{~s}$ ?
(b) At $t=0.9 \mathrm{~s}$, the switch is flipped back to the right. Immediately after the flip, what is the current through $R$ ?
(c) What is the voltage across $R$ at time $t=0.91 \mathrm{~s}$ ?

( $\left.10^{\text {pts }}\right)$ 8. In lectures and labs we studied the R-S flip-flop built from NAND gates. Consider the flip-flop circuit below constructed from NOR gates. Fully describe the operation of this circuit. In particular, what are the states of $Q$ and $\bar{Q}$ when $R$ was last HI? What about if $S$ was last HI? Justify your answers to receive full credit. In the circuit below, lines that cross without a dot are not connected.


